

## Summer Review for Math 124 (Calculus I Part 2)

Note: This packet is a graded assignment due at week 1, detailed work/steps are required for full credit.  
The contents will be tested at beginning of the course.

Name \_\_\_\_\_

**SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.**

**Find the limit.**

1)  $\lim_{x \rightarrow 0} \frac{x^3 - 6x + 8}{x - 2}$  1) \_\_\_\_\_

2)  $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x}$  2) \_\_\_\_\_

**Provide an appropriate response.**

3) It can be shown that the inequalities  $-x \leq x \cos\left(\frac{1}{x}\right) \leq x$  hold for all values of  $x \geq 0$ . 3) \_\_\_\_\_

Find  $\lim_{x \rightarrow 0} x \cos\left(\frac{1}{x}\right)$  if it exists.

4) The inequality  $1 - \frac{x^2}{2} < \frac{\sin x}{x} < 1$  holds when  $x$  is measured in radians and  $|x| < 1$ . 4) \_\_\_\_\_

Find  $\lim_{x \rightarrow 0} \frac{\sin x}{x}$  if it exists.

**Find the limit.**

5)  $\lim_{x \rightarrow 1^-} \frac{x^2 - 4x + 3}{x^3 - x}$  5) \_\_\_\_\_

6)  $\lim_{x \rightarrow (-\pi/2)^-} \sec x$

6) \_\_\_\_\_

**Divide numerator and denominator by the highest power of x in the denominator to find the limit.**

7)  $\lim_{x \rightarrow \infty} \sqrt{\frac{25x^2}{6 + 16x^2}}$

7) \_\_\_\_\_

8)  $\lim_{x \rightarrow \infty} \frac{5x + 6}{\sqrt{6x^2 + 1}}$

8) \_\_\_\_\_

**Provide an appropriate response.**

9) Use the Intermediate Value Theorem to prove that  $x(x - 2)^2 = 2$  has a solution between 1 and 3.

9) \_\_\_\_\_

**Find numbers a and b, or k, so that f is continuous at every point.**

10)

$$f(x) = \begin{cases} -7, & x < -4 \\ ax + b, & -4 \leq x \leq 3 \\ 21, & x > 3 \end{cases}$$

10) \_\_\_\_\_

11)

$$f(x) = \begin{cases} x^2, & \text{if } x \leq 2 \\ kx, & \text{if } x > 2 \end{cases}$$

11) \_\_\_\_\_

The function  $s = f(t)$  gives the position of a body moving on a coordinate line, with  $s$  in meters and  $t$  in seconds.

12)  $s = 9t - t^2, 0 \leq t \leq 9$

Find the body's displacement and average velocity for the given time interval.

12) \_\_\_\_\_

**Solve the problem.**

13) At time  $t$ , the position of a body moving along the  $s$ -axis is  $s = t^3 - 21t^2 + 120t$  m. Find the body's acceleration each time the velocity is zero.

13) \_\_\_\_\_

14) Suppose that the dollar cost of producing  $x$  radios is  $c(x) = 200 + 10x - 0.2x^2$ . Find the average cost per radio of producing the first 50 radios.

14) \_\_\_\_\_

**Find  $dy/dt$ .**

15)  $y = \cos^4(\pi t - 17)$

15) \_\_\_\_\_

16)  $y = 5t(2t + 4)^4$

16) \_\_\_\_\_

**Use implicit differentiation to find  $dy/dx$  and  $d^2y/dx^2$ .**

17)  $xy - x + y = 6$

17) \_\_\_\_\_

**At the given point, find the line that is normal to the curve at the given point.**

18)  $x^6y^6 = 64$ , normal at (2, 1)

18) \_\_\_\_\_

**Find the derivative of y with respect to the independent variable.**

19)  $y = 2\sqrt{t}$

19) \_\_\_\_\_

**Find the indicated tangent line.**

20) Find the tangent line to the graph of  $f(x) = -8e^{5x}$  at the point (0, -8).

20) \_\_\_\_\_

**Use logarithmic differentiation to find the derivative of y with respect to the independent variable.**

21)  $y = (x + 10)^x$

21) \_\_\_\_\_

**Find the formula for  $df^{-1}/dx$ .**

22)  $f(x) = x^{5/3}$

22) \_\_\_\_\_

**Solve the problem.**

23) A wheel with radius 3 m rolls at 15 rad/s. How fast is a point on the rim of the wheel rising when the point is  $\pi/3$  radians above the horizontal (and rising)? (Round your answer to one decimal place.)

23) \_\_\_\_\_

24) A piece of land is shaped like a right triangle. Two people start at the right angle of the triangle at the same time, and walk at the same speed along different legs of the triangle. If the area formed by the positions of the two people and their starting point (the right angle) is changing at  $2 \text{ m}^2/\text{s}$ , then how fast are the people moving when they are 3 m from the right angle? (Round your answer to two decimal places.)

24) \_\_\_\_\_

**Solve the problem. Round your answer, if appropriate.**

25) Boyle's law states that if the temperature of a gas remains constant, then  $PV = c$ , where  $P$  = pressure,  $V$  = volume, and  $c$  is a constant. Given a quantity of gas at constant temperature, if  $V$  is decreasing at a rate of  $15 \text{ in.}^3/\text{sec}$ , at what rate is  $P$  increasing when  $P = 40 \text{ lb/in.}^2$  and  $V = 80 \text{ in.}^3$ ? (Do not round your answer.)

25) \_\_\_\_\_

## Answer Key

Testname: SUMMER\_PACKET FOR MATH124\_CALCULUS1B

- 1) -4
- 2)  $1/2$
- 3) 0
- 4) 1
- 5) - 1
- 6)  $\infty$
- 7)  $\frac{5}{4}$
- 8)  $\frac{5}{\sqrt{6}}$
- 9) Let  $f(x) = x(x - 2)^2$  and let  $y_0 = 2$ .  $f(1) = 1$  and  $f(3) = 3$ . Since  $f$  is continuous on  $[1, 3]$  and since  $y_0 = 2$  is between  $f(1)$  and  $f(3)$ , by the Intermediate Value Theorem, there exists a  $c$  in the interval  $(1, 3)$  with the property that  $f(c) = 2$ . Such a  $c$  is a solution to the equation  $x(x - 2)^2 = 2$ .
- 10)  $a = 4, b = 9$
- 11)  $k = 2$
- 12) 0 m, 0 m/sec
- 13)  $a(10) = 18 \text{ m/sec}^2, a(4) = -18 \text{ m/sec}^2$
- 14) \$4.00
- 15)  $-4\pi \cos^3(\pi t - 17) \sin(\pi t - 17)$
- 16)  $5(2t + 4)^3(10t + 4)$
- 17)  $\frac{dy}{dx} = \frac{1 - y}{1 + x}; \frac{d^2y}{dx^2} = \frac{2y - 2}{(x + 1)^2}$
- 18)  $y = 2x - 3$
- 19)  $\frac{\ln 2}{2\sqrt{t}} 2\sqrt{t}$
- 20)  $y = -40x - 8$
- 21)  $(x + 10)^x \left( \ln(x + 10) + \frac{x}{x + 10} \right)$
- 22)  $\frac{3}{5}x^{-2/5}$
- 23) 22.5 m/s
- 24) 0.67 m/s
- 25)  $\frac{15}{2} \text{ lb/in.}^2 \text{ per sec}$